# Some comments concerning the analytic solution for the model of a homogeneous halfspace

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## 1 Analytic solution

The problem of a vertical single force acting on the surface of a homogeneous halfspace and its solution was firstly discussed by Lamb (1904). Later discussions of this problem can be found for example in Ewing et al. (1957) or Forbriger (1996). Therefore, I will just give here the final equations for displacements in polar coordinates $r$, $\varphi$ and $z$ (see eq. (1.47) in Forbriger (1996)) which are used for the calculation of the semi-analytic seismograms in this benchmark test:

$$u_r(r, z=0, \omega, t) = \frac{F_0}{2\pi\mu}\,\omega\, e^{i\omega t} \int_0^{+\infty} \frac{u^2}{D(u)} \left(u^2 - b^2 + 2ab\right) J_1(u\omega r)\,\mathrm{d}u \tag{1.1a}$$

$$u_\varphi(r, z=0, \omega, t) = 0 \tag{1.1b}$$

$$u_z(r, z=0, \omega, t) = -\frac{F_0}{2\pi\mu}\,\omega\, e^{i\omega t} \frac{1}{\beta^2} \int_0^{+\infty} \frac{uia}{D(u)} J_0(u\omega r)\,\mathrm{d}u \tag{1.1c}$$

with

$$D(u) = \left(u^2 - b^2\right)^2 + 4u^2 ab. \tag{1.2}$$

In the previous equations $\omega$ is the angular frequency, $a$ is the vertical slowness for P-waves, $b$ is the vertical slowness for S-waves, $u$ is the horizontal component of the slowness, $F_0$ is the force time function in N acting on the surface, $\mu$ is the shear modulus and $J_0$ and $J_1$ are the Bessel functions of the first kind of order zero and one, respectively.

The slowness components $a$, $b$ and $u$ are linked by the following equations:

$$a^2 + u^2 = s_\alpha^2 = \frac{1}{\alpha^2} \tag{1.3a}$$

$$b^2 + u^2 = s_\beta^2 = \frac{1}{\beta^2} \tag{1.3b}$$

$$\tag{1.3c}$$

# 2 Numerical calculation

The slowness integrals in eq. (1.1) are approximated by the trapezoidal rule in the numerical calculation. To decrease the amplitude of the cutoff phase the integrands are multiplied with a slowness dependent taper. (The cutoff phase occurs because the integration can only be done up to a finite value and not up to infinity.) The cutoff phase mainly disturbs the waveforms in the near offset range.

In the acquisition geometry used in the TOAST benchmark tests for shallow seismics the radial component $r$ corresponds to the $x$ component and the vertical components $z$ coincide.

To obtain particle velocities instead of displacements one simply has to use the first time derivative of the force time function for the calculation.

# References


Ewing, W., Jardetzky, W. & Press, F., 1957, Elastic waves in layered media, McGraw-Hill Book Company, Inc., New York–Toronto–London.

Forbriger, T., 1996, Interpretation von Oberflächenwellen in der Flachseismik, Diplomarbeit, Institut für Geophysik, Universität Stuttgart.
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Lamb, H., 1904, On the propagation of tremors over the surface of an elastic solid, Phil. Trans. R. Astr. Soc., 203, 1–42.