Involving `sofi2D` in TOAST benchmark test

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February 21, 2014

1 Introduction

`sofi2D` is a viscoelastic 2D FD forward modeling code. To include also this code into the TOAST benchmark test a 2D to 3D transformation suggested by Amundsen & Reitan (1994) is applied to the modeled data to make them comparable with the results of the 3D forward modeling codes.
In the following I will just briefly rewrite the relevant equations for this transformation and make some comments on the numerical implementation of the transformation.

2 2D to 3D transformation

In 2D calculations the model parameters as well as the sources are implicitly assumed to be extended to infinity in the third dimension. Therefore, 2D sources correspond to line sources in 3D.

In the TOAST benchmark tests the free surface coincides with the plane z=0 and the vertical source as well as the receivers are located at the free surface. Assuming a line source located along the y-axis the 2D wavefields for the vertical component z and the horizontal component x can be expressed by

$$u_z^{2D}(x,\omega) = 2\int_0^\infty \omega\cos(\omega p x)\, G_z(\omega,p)\,\mathrm{d}p \tag{2.1a}$$

$$u_x^{2D}(x,\omega) = 2\int_0^\infty \omega\sin(\omega p x)\, G_x(\omega,p)\,\mathrm{d}p \tag{2.1b}$$

with the distance x to the line source, the slowness p, the angular frequency ω and the expansion coefficients G_z and G_x for the vertical and the horizontal component respectively.

The corresponding back transformation can be written as

$$G_z(\omega,p) = \frac{1}{\pi}\int_0^\infty \cos(\omega p x)\, u_z^{2D}(x,\omega)\,\mathrm{d}x \tag{2.2a}$$

$$G_x(\omega,p) = \frac{1}{\pi}\int_0^\infty \sin(\omega p x)\, u_x^{2D}(x,\omega)\,\mathrm{d}x \tag{2.2b}$$

and is used during the transformation to calculate the expansion coefficients G_x and G_z.

To obtain the 3D wavefields one has to do an expansion with Bessel functions to obtain

$$u_z^{3D}(r,\omega) = \int_0^\infty \omega^2 p J_0(\omega p r)\, G_z(\omega,p)\,\mathrm{d}p \tag{2.3a}$$

$$u_x^{3D}(r,\omega) = \int_0^\infty \omega^2 p J_1(\omega p r)\, G_x(\omega,p)\,\mathrm{d}p \tag{2.3b}$$

with J_0 and J_1 as Bessel functions of the first kind of order zero and one respectively.

For a cylindrical symmetric source (such as a vertical force) and a horizontally layered medium (1D medium) this transformation is analytically exact. However, in the numerical calculations the integrals have to be solved numerically and therefore they are approximated by discretization.

3 Numerical implementation

In the numerical implementation the integrals in equations (2.2) and (2.3) are solved by the trapezoidal rule.

3.1 Calculation of the expansion coefficients

3.1.1 Shallow seismic benchmark tests

To reduce spatial aliasing I model a 2D dataset with 436 traces. The minimum offset is 1 m, the maximum offset is 88 m and an equidistant receiver spacing of 0.2 m is used. These seismograms are used to calculate the expansion coefficients G_z and G_x according to equation (2.2). For the calculation I apply an offset dependent cosine taper to the far offset traces. 10 % of the traces (farest away from the source) are affected by this taper.

3.1.2 Ultrasound benchmark tests

For the ultrasound benchmark tests I model a 2D dataset with 203 traces. The minimum offset is 3 mm, the maximum offset is 205 mm and I use an equidistant receiver spacing of 1 mm. Again an offset dependent cosine taper is applied to the far offset traces during the calculation of the expansion coefficients G_z and G_x. This taper affects 10 % of the traces (farest away from the source).

3.2 Calculation of the 3D seismograms

Equation (2.3) is used to calculate the 3D seismograms. To reduce the influence of the cutoff phase a slowness taper is applied here. For the shallow seismic benchmark tests the slowness integral is calculated up to a slowness of 12.0 s/km but from 8.0 s/km on a cosine taper is applied. For the ultrasound benchmark tests the limits for the slowness integral vary for the different benchmark tests.

3.3 Calculation of the seismograms for model 3 for shallow seismics

For the gradient model the seismograms are not decayed completely after 0.7 s. Therefore, I calculated longer 2D timeseries for the transformation (up to 1.0 s) to avoid artefacts during the FFTs calculated during the 2D to 3D transformation.

References

Amundsen, L., Reitan, A., 1994, Transformation from 2-D to 3-D wave propagation for horizontally layered media, *Geophysics*, **59**(12), 1920–1926.