Geophysical Institute
Project TOAST
(Toolbox for Applied Seismic Tomography)

Appropriate line source simulation procedure for shallow seismic field data

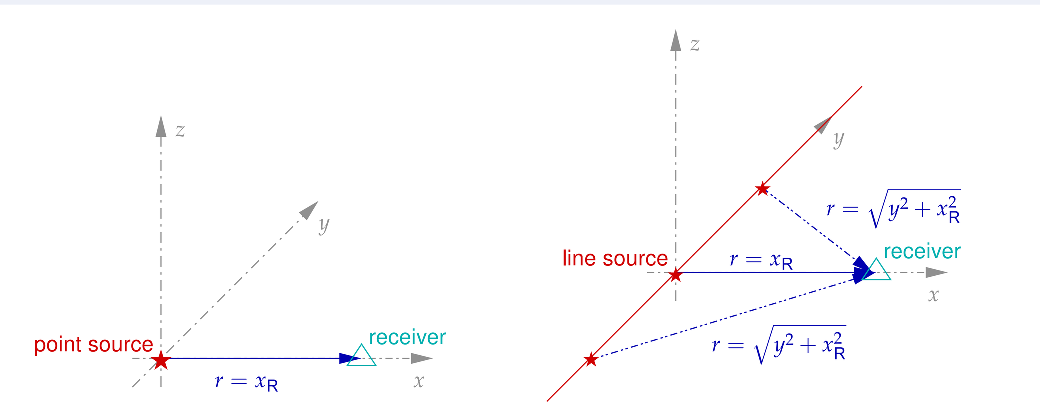
Thomas Forbriger, Lisa Groos, Martin Schäfer

1. Statement of the problem

Shallow seismic field data is excited by point sources (e.g. hammer blows). Full waveform inversion (FWI) approaches which make use of 2D forward modelling implicitely use a line source to fit the observed data. Therefore recorded waveforms must be transformed to simulate equivalent line source generated data prior to application of 2D FWI.

Different approaches are known from literature. They are primarily developed for reflected waves and are not appropriate for shallow seismic data. They fail on 2D heterogeneous structures or provide inappropriate amplitude scaling. We present a simple and effective procedure to transform shallow seismic data.

2. Definition of the line source



A line source can be understood as a continuous arrangement of superimposed point sources.

The wave's energy spreads on the surface of a sphere (amplitude decay $\propto 1/r$) or a cylinder (amplitude decay $\propto 1/\sqrt{r}$) if radiated from a point source or a line source, respectively.

While the transient point source produces a transient displacement at the receiver in 3D full-space, this is not the case for the line source. The displacement time history produced by the line source has a sharp onset and decays slowly to zero as point sources of larger offset along y contribute to the signal at larger travel time.

The displacement vector field is given by

$$\mathbf{u}(\mathbf{x},t) = \int_{-\infty}^{+\infty} \iiint_V \underline{\mathbf{G}}(\mathbf{x},t;\mathbf{x}',t')\,\mathbf{f}(\mathbf{x}',t')\;\mathrm{d}^3\mathbf{x}'\,\mathrm{d}\,t', \tag{1}$$

where $\underline{\mathbf{G}}(\mathbf{x},t;\mathbf{x}',t')$ is Green's tensor and $\mathbf{f}(\mathbf{x},t)$ defines the seismic source. The force density for a point source at $\mathbf{x}_\mathrm{S}$ is

$$\mathbf{f}_\mathrm{P}(\mathbf{x},t;\mathbf{x}_\mathrm{S}) = \mathbf{F}(t)\,\delta_x(x-x_\mathrm{S})\,\delta_x(y-y_\mathrm{S})\,\delta_x(z-z_\mathrm{S}) \text{ and} \tag{2}$$

$$\mathbf{f}_\mathrm{L}(x,y,z,t;x_\mathrm{S},z_\mathrm{S}) = \mathbf{F}(t)\,\mathcal{C}\,\delta_x(x-x_\mathrm{S})\,\delta_x(z-z_\mathrm{S}) \tag{3}$$

for a line source with Dirac's delta function $\delta_x(x)$ of $[\delta_x(x)] = 1\,\mathrm{m}^{-1}$, $[\mathbf{F}] = 1\,\mathrm{N}$, and $\mathcal{C} = 1\,\mathrm{m}^{-1}$. For a 2D structure $\underline{\mathbf{G}}(\mathbf{x},t;\mathbf{x}',t')$ is constant in y and hence the equivalent displacement field of a line source

$$\mathbf{u}_\mathrm{L}(x,y,z,t;x_\mathrm{S},z_\mathrm{S}) = \int_{-\infty}^{+\infty} \mathbf{u}_\mathrm{P}(x,y,z,t;x_\mathrm{S},y',z_\mathrm{S})\,\mathcal{C}\;\mathrm{d}\,y'. \tag{4}$$

can be obtained from the displacement field generated by a point source by integration along y.

3. Fourier-Bessel transformation

The Fourier-Bessel expansion coefficients C_z and C_r can be obtained from point source data recorded on 1D structures by inversion of

$$\tilde{u}_{\mathrm{P}z}(r,t;z_\mathrm{S}) = \int_0^{+\infty} C_z(k,\omega;z_\mathrm{S})\,J_0(kr)\,k\;\mathrm{d}\,k \text{ and} \tag{5}$$

$$\tilde{u}_{\mathrm{P}r}(r,t;z_\mathrm{S}) = \int_0^{+\infty} C_r(k,\omega;z_\mathrm{S})\,J_1(kr)\,k\;\mathrm{d}\,k. \tag{6}$$

The Fourier-Bessel transformation of the vertical and horizontal component of displacement then is obtained by

$$\tilde{u}_{\mathrm{L}z}(x,\omega;z_\mathrm{S}) = \int_0^{+\infty} C_z(k,\omega;z_\mathrm{S})\,2\,\cos(kx)\,\mathcal{C}\;\mathrm{d}\,k \text{ and} \tag{7}$$

$$\tilde{u}_{\mathrm{L}x}(x,t;z_\mathrm{S}) = \int_0^{+\infty} C_r(k,\omega;z_\mathrm{S})\,2\,\sin(kx)\,\mathcal{C}\;\mathrm{d}\,k. \tag{8}$$

for the equivalent line source. While this approach is mathematically exact for 1D structures it suffers from cut-off effects when being applied to recorded data and fails for waves recorded on structures with significant 2D heterogeneity.

4. Solution of the acoustic wave equation

3D Green's function in full-space (point source):

$$G_k^{3\mathrm{D}}(\mathbf{x};\mathbf{x}_\mathrm{S},k) = g_k^{3\mathrm{D}}(R;k) = \frac{e^{ikR}}{R} \tag{9}$$

$R = |\mathbf{x} - \mathbf{x}_\mathrm{S}|$ is the source to receiver offset.

2D Green's function in full-space (line source):

$$G_k^{2\mathrm{D}}(\mathbf{x};\mathbf{x}_\mathrm{S},k) = g_k^{2\mathrm{D}}(R;k) = i\mathcal{C}\,\pi\,H_0^{(1)}(kR) \tag{10}$$

Far-field approximation of 2D solution (line source):

$$\lim_{R\to\infty} g_k^{2\mathrm{D}}(R;k) \to \sqrt{\frac{2\pi}{kR}}\,e^{ikR}\,e^{i\pi/4}\,\mathcal{C} \tag{11}$$

Far-field simulation of line source displacement:

$$\lim_{r\to\infty} \tilde{u}_\mathrm{L}(x=r,\omega) = F_{\text{-D}}(r,k)\,\tilde{u}_\mathrm{P}(r,\omega) \tag{12}$$

Factor derived from acoustic wave Green's function:

$$F_{\text{-D}}(r,k) = \lim_{r\to\infty} \frac{g_k^{2\mathrm{D}}(r)}{g_k^{3\mathrm{D}}(r)} = \sqrt{\frac{2\pi r}{k}}\,e^{i\pi/4}\,\mathcal{C} \tag{13}$$

$F_{\text{-D}}(r,k)$ can also be motivated by (a) $g_k^{1\mathrm{D}}(r)/g_k^{2\mathrm{D}}(r)$, (b) expansion kernels in eqs. (5) to (8), and (c) S-wave excited by hammer blow.

5. Single-trace transformation procedures

Single velocity transformation for phase velocity $v_\mathrm{ph} = \omega/k$:

$$F_{\text{-D}}(r,k) = \sqrt{\frac{2\pi r}{k}}\,e^{i\pi/4}\,\mathcal{C} = \underbrace{\sqrt{2rv_\mathrm{ph}}}_{F_\mathrm{amp}} \cdot \underbrace{\sqrt{\frac{\pi}{\omega}}\,e^{i\pi/4}}_{\tilde{F}_{\sqrt{t^{-1}}}(\omega)} \cdot\mathcal{C} \tag{14}$$

$\tilde{F}_{\sqrt{t^{-1}}}(\omega)$ is the Fourier transform of

$$F_{\sqrt{t^{-1}}}(t) = \begin{cases} \frac{1}{\sqrt{t}} & \text{if } t \geq 0 \text{ and} \\ 0 & \text{else.} \end{cases} \tag{15}$$

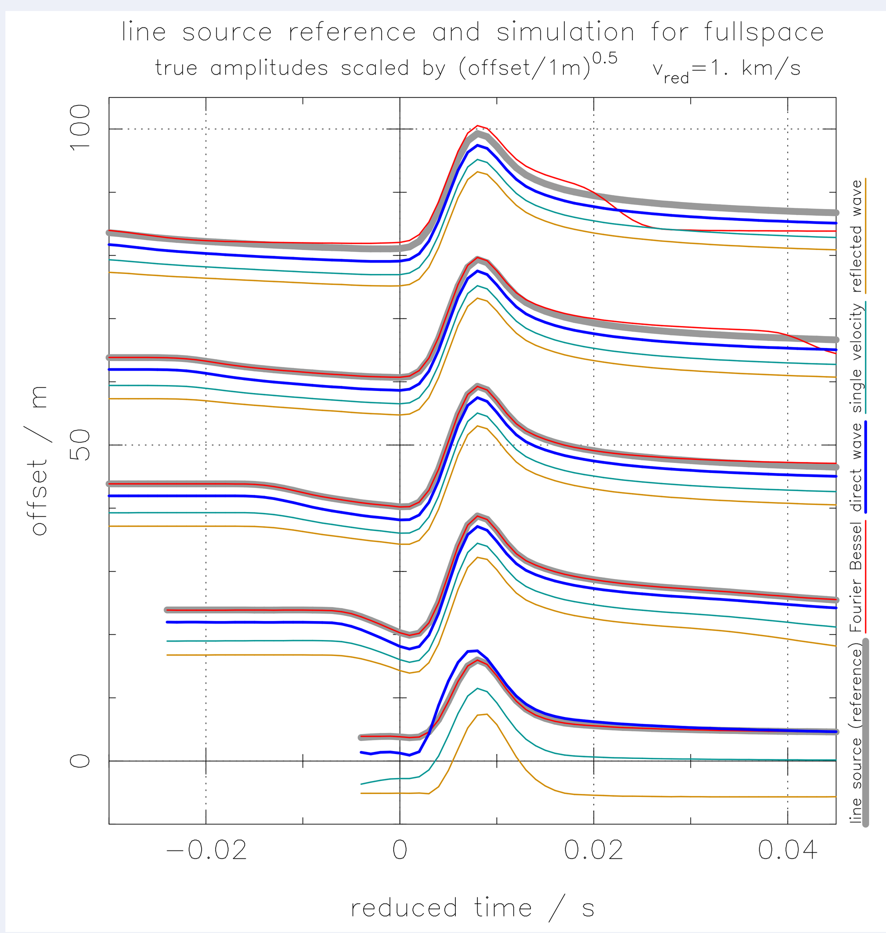
Reflected wave transformation:
(v_ph known *a priori* and travel distance $r = t\,v_\mathrm{ph}$ from sample time t)

$$F_\mathrm{amp} = v_\mathrm{ph}\,\sqrt{2t} \tag{16}$$

Direct wave (shallow seismics) transformation:
(travel distance equals offset r and $v_\mathrm{ph} = r/t$ from sample time t)

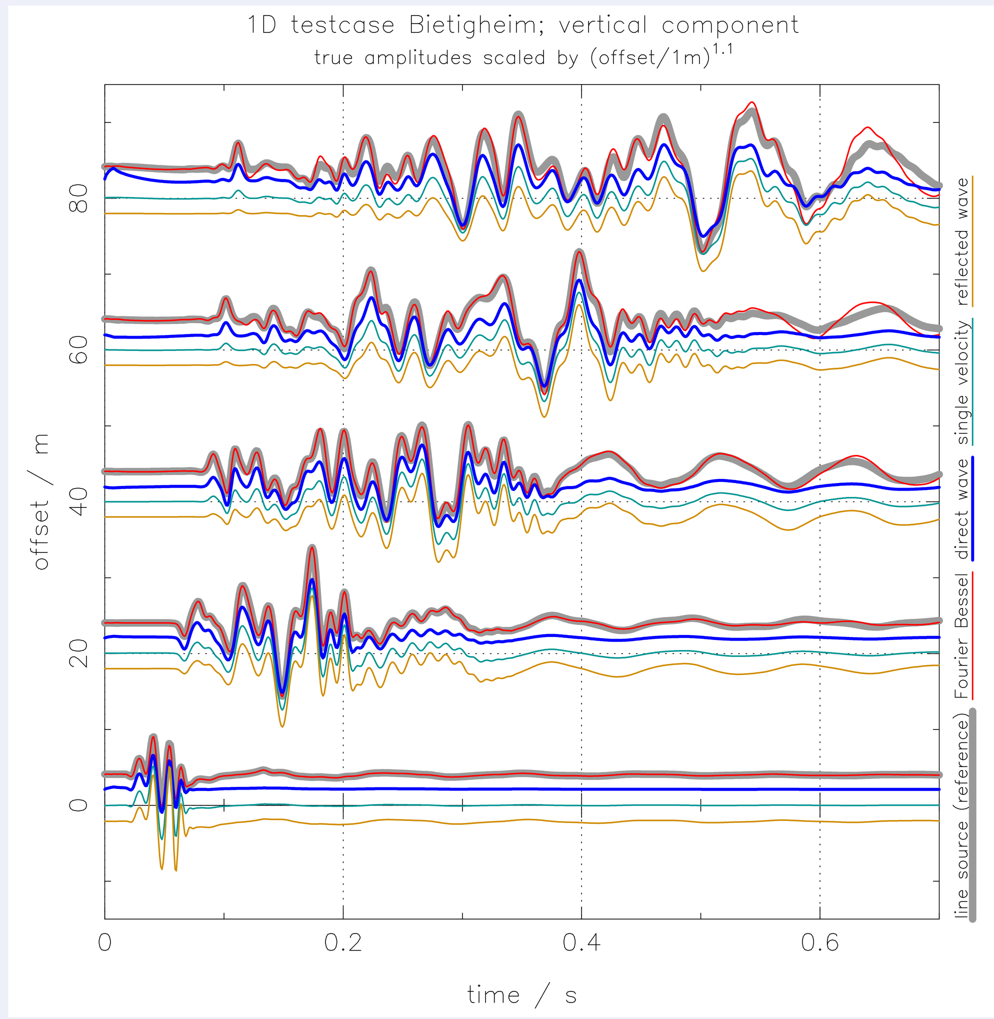
$$F_\mathrm{amp} = r\,\sqrt{\frac{2}{t}} = r\,\sqrt{2}\,F_{\sqrt{t^{-1}}}(t) \tag{17}$$

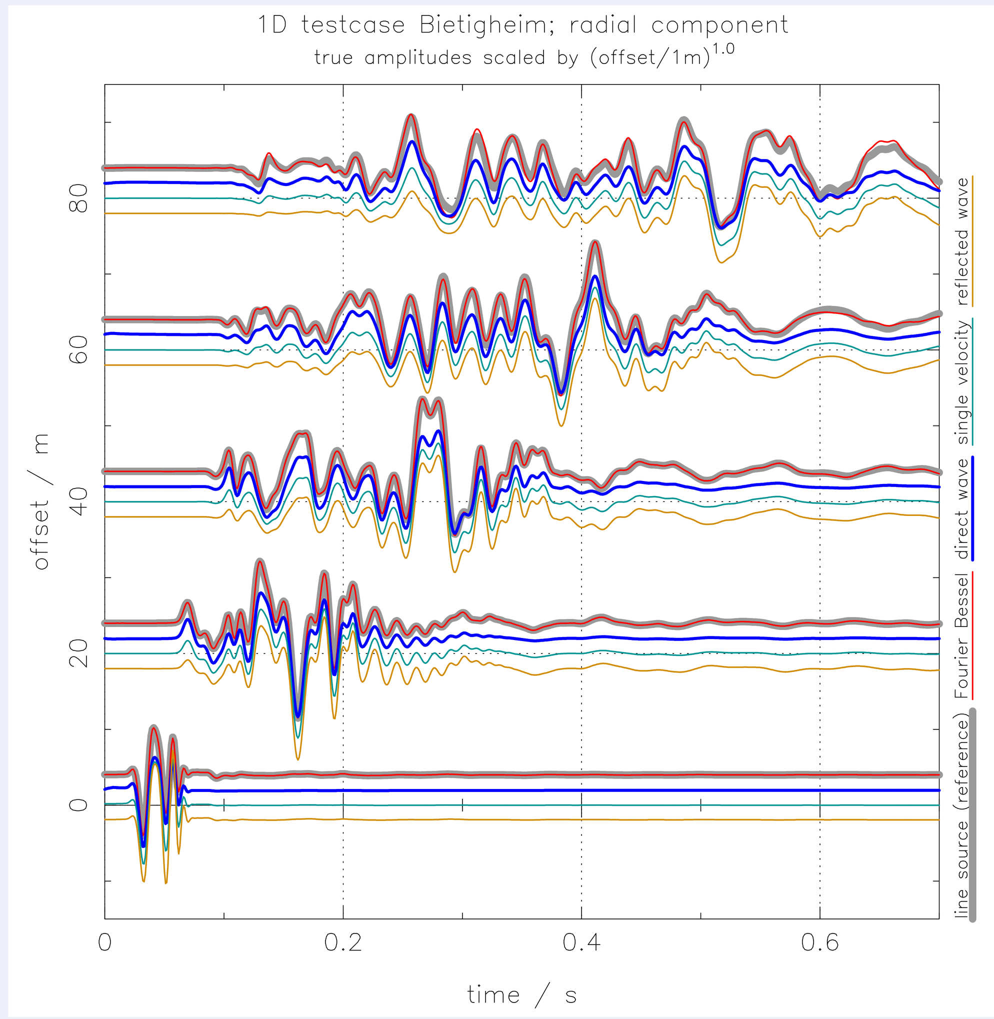
6. Homogeneous full-space

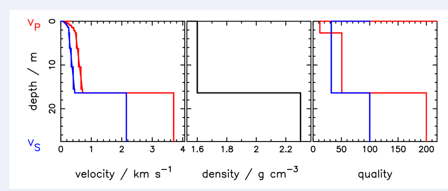


For reference we compare exact and simulated line source seismograms in homogeneous full-space where all approaches should provide identical far-field simulations. Reflectivity seismograms for point- and line-source are calculated in 3D full-space with $v_S = 1\,\mathrm{km\,s^{-1}}$, $v_P = 1.7\,\mathrm{km\,s^{-1}}$, $\rho = 1.8 \cdot 10^3\,\mathrm{kg\,m^{-3}}$, and $Q_P = Q_S = 100$.

7. Layered half-space







Reflectivity seismograms for a strongly dispersive subsurface model obtained by inversion of shallow seismic field data.

Simulated line source seismograms are obtained by transformation procedures from point source seismograms. Line source reference seismograms are obtained by the reflectivity method. Waveforms are displayed with true amplitudes scaled by an offset dependent factor. Only the Fourier-Bessel and the direct wave transformation are able to produce correct amplitudes for waves of any propagation velocity.
2D cases are presented by Schäfer *et al.* (S1-6-002).

8. Conclusions

Equivalent line source data can be simulated from shallow seismic point source recordings by a simple but effective prodecure:

1. scale waveform by $r\sqrt{2}$ (offset times square root of 2)
2. convolve with $1/\sqrt{t}$ (fractional half integration)
3. taper samples with $1/\sqrt{t}$

An implementation of the concepts discussed here is available in the program `lisousi` which is provided at `http://www.opentoast.de`.

9. Acknowledgements

This research was carried out within the TOAST project. The TOAST project is supported by the Federal Ministry of Education and Research grant 03G0752A within the GEOTECHNOLOGIEN programme.